An essential calculation which had to be completed before moving on to other analyses is that of the linkage lengths (most importantly Link1 and Link 2) and their corresponding “workable” area. In this analysis, we try to maximize the link lengths to maximize the workable area which can be sculpted using the tool bit. At the same time, the link sizes should not be too large in order to minimize size and weight of the overall device. Thus, a balance must be struck between link sizes and workable area. The following user satisfaction curves illustrate the relative importance of each:

Cannot reach most critical points on a normal-sized knee

Can reach most critical points of a normal-sized knee

Can reach all points on a large-sized knee

From the graphs, it can be seen that the link weights are slightly more important for user satisfaction, and thus, the structural design focus will be to reduce link size (and weight) more than to achieve a larger workable area than what’s functionally needed.

This section of the report explains the theory behind the link size calculations and how we determined them to be within discrete ranges. First of all we must define how we express the workable area at the tool bit. From the *Matlab* plots results discussed in the CFP, we already know the general shape of these areas. They can be *approximated* as the area between two circular boundaries, one boundary of which is larger than the other. Thus, we define the outer or larger curved boundary as Rmax and the inner our smaller curved boundary as Rmin. The side boundaries of the workable area are approximated as straight vertical lines, and the distance between them is called Rangex. Thus, the workable area looks like the following (note: in reality, the origins of the radii should be different, but this is a very close approximation):

Rmin

Rmax

Rangex

With the workspace coordinates defined, we must also define the variables of the linkage design in order to optimize the linkage sizes. The linkage variables are vital for the analysis and are defined in the figure below. The figure is a simplified illustration of the linkage orientation, with the square point representing the bone mount and the circular point representing the cutting tool. The legend to the left identifies the link names.

L3

LR

L5

θ3-5

θ5

R

Bone Mount

Tool

Bit

**LR:** Radial Link

**L3:** Vertical Offset Link

**L5:** Tool Link (resultant length)

y

z

With the configuration above, we need to maximize the workable area while, at the same time, minimizing link lengths. The goal of the following analysis is to optimize the link lengths and workable area. We perform the analysis by first computing a mathematical relationship of the workable area based on the link lengths and angles defined in the figure above.

To start, we write the following geometric relationships between distances and angles:

(note: L1 , L2 and θ12 are not shown in the diagram)

We will create a virtual length called L’5:

L’5 = L5cosθ5

With that new virtual length, we can define the “reachable” length in the “z” and “y” directions stated above:

90 - )

90 - )

Since the “z” and “y” lengths described above are components of the R vector, we can compute the length of R simply as follows:

R is the range vector which characterizes the size of the workable area in our analysis.

Knowing the mathematical relationship between link lengths (sizes) and workable area, our task is to find an optimized combination, which gives us both a satisfactory workable area and a reasonable size and weight for the device. We do so with an excel spreadsheet. We insert the variables and equations above into the spreadsheet, and observe the change in workable area based on links 1 and 2. It is known that the workable area increases with increasing link sizes, however, our goal was to pinpoint the “plateau” of the relationship. The spreadsheet allowed us to quickly and accurately calculate the minimum and maximum radius values, along with the full range, with different combinations of theta values. An example is given below.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Link** | **Length** |  |  | **Theta\_12\_min** | 30 |
| 1 | 7 |  |  | **Theta\_12\_max** | 120 |
| 2 | 7 |  |  | **Theta\_5\_max** | 45 |
| 3 | 5 |  |  | **Theta\_3** | 20 |
| 4 | 5 |  |  | **Theta\_35** | -15 |
| 5 | 7 |  |  |  |  |
|  |  |  |  |  |  |
| **r\_min** | 1.740303 |  |  |  |  |
| **r\_max** | 9.910691 |  |  |  |  |
| **Range\_x** | 9.899495 |  |  |  |  |

By inserting various combinations of link sizes into the spreadsheet, we observed a preliminary viable option. It gave us a large workable area while not overly extending the length of the links, and happened when both links 1 and 2 are equal in length and measure 7 cm. Our next task was to perform a sensitivity analysis and verification of this preliminary result, and we did that using Matlab.

First, we plotted the combined weight of 7 links versus the length of each link. This is done by assuming a rectangular slab for each link and the following formula:



Figure : Weight (lb) vs. Link Length (cm)

Based on the user satisfaction curve vs. the device total weight, we should not go beyond 5 lb for the device as a whole. Thus, the links alone, without the motors, encoders and other equipment, should not be more than approximately 2 lb – corresponding to around a 7 cm link length. This verifies that the preliminary result of a 7 cm link size does not conflict our weight requirements. However, we must still check whether it conflicts with our workable area requirements. To check that the link lengths which fall below the maximum acceptable weight also give us an acceptable workable area, we plot the mathematical range relationship in Matlab. The following workable area relations are entered:

L’5 = L5cosθ5

90 - )

90 - )

Plotting the workable area range versus the link size, we get the following graph. On the plot, a link size of 7 cm corresponds to a workable range of approximately 90 mm, or 9 cm. This value is greater than the standard implant size that we aim to achieve, thus, a link size of 7 cm is both acceptable for maximum weight constraints and minimum workable area constraints.



The Matlab code for the analysis described above can be found in the Appendix.

*>> y = x.^2 + x.^2 -2.\*x.\*x.\*cos(30) + 5\*cos(20) - 7\*cos(45)\*cos(90-20+15);*

*>> z = y;*

*>> y = -5\*cos(20) + 7\*cos(45)\*cos(90-20+15);*

*>> R = sqrt(z.^2 + y^2);*

*>> plot(x,R)*

*>> density = 8.03;*

*>> conversion = .0022046;*

*>> weight = x\*2\*1\*density\*conversion;*

*>> f = 7.\*weight;*

*>> plot(x,f)*

*>> plot(x,f)*